EFFECT OF CONFIGURATION OF A CONFINED SPACE ON THE PARAMETERS OF CONVECTIVE FLOWS FOR SYMMETRIC TEMPERATURE FIELDS

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UDC 532.529.2

We present a simplified method for calculating the parameters of convective flows in a confined space of various configurations.

In recent years, a large number of works has been devoted to the determination of parameters of convective heat exchange between a solid surface and the adjacent liquid and gas. However, the derived test dependences for the vertical [1] and horizontal [2] surfaces are not realized in conditions of confined volumes, as is convincingly shown in [3]. The most complete results on the heat exchange in a confined volume are given in [4].

The calculation of parameters of free convective flows in a confined space up to now has presented considerable difficulties. The classical formulation of the internal problem for the convective motion of a liquid or gas is distinguished by extreme complexity [5, 6] and the obtained solutions are often unsuitable for simple engineering calculations. Therefore, it is undoubtedly of interest to develop a simplified, but practically useful, method of calculating the parameters of free convective flows in a confined space. In a simplified method of calculation, one may use the theoretical methods [7] for the infinite space and introduce necessary changes and additions which are associated with various configurations of the confined volume [in the horizontal cross section, these are square, rectangle, or circle (Fig. 1)], in conditions of a symmetric temperature field, i.e., when the temperatures of the walls are equal.

Below we make the following assumptions:

1) The cover (ceiling) of the confined volume does not have a significant effect (thermal or aerodynamic) on the process.

2) The velocity field of the reverse flow (passive reverse flow) is uniform in the transverse section.

3) The boundary where the convective flow changes direction (sign) is infinitely thin, i.e., in this region, the viscosity of the gas is close to zero, and the friction forces are absent.

4) The distribution of the heat flux from the field is uniform in the whole volume of the moving gas.

5) The closed volume is assumed to be aerodynamically insulated from the surrounding space.

6) The velocity and temperature fields in the cross section of the active flow (which is adjacent to the side walls whose temperature is different from that of the ceiling) correspond to those adopted in [7]. For the velocity fields, the sticking zone of the flow next to the wall is neglected on account of its relative smallness:

$$w_z = w_{zm} \exp\left[-\frac{1}{2}\left(\frac{y}{cz}\right)^2\right] \quad \text{for} \quad y > 0, \tag{1}$$

$$\Theta_z = \Theta_{zm} \exp\left[-\frac{\sigma}{2}\left(\frac{y}{cz}\right)^2\right]; \tag{2}$$

Belorussian Polytechnic Institute, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 43, No. 4, pp. 554-559, October, 1982. Original article submitted October 6, 1981.

0022-0841/82/4304-1073$07.50

7) The gas concentration is distributed uniformly everywhere, i.e., the gravitation component is due only to the nonuniformity of the temperature field.

8) The dimension (width) of the zone where the gas flow turns away from a vertical wall to the field is equal to the (maximum) width of the active part of the convective flow.

The temperature of the reverse flow $T_z^{(-)}$ at the level z can be determined as a sum of the average temperature of the downward flow at the level of the field ($T_{z,av}$) and the temperature increase of the air on account of the heat transfer from the field to the flow ($T_{f,av}$):

$$T_z^{(-)} = T_{z\,av} + \Delta T_{f.av}, \tag{3}$$

where

$$T_{z\,av} = T_z^{(-)} = \frac{\int_0^{b-\Delta b_z} \Theta_z dy}{b - \Delta b_z}; \tag{4}$$

$$\Delta T_{f.av} = \frac{\alpha_f (T_f - T_{z.av})\, b\,(b - \Delta b_z)}{2 c_p \rho V_f^2}. \tag{5}$$

Expression (4) contains the excess temperature in the transverse section of the active flow. Using [7] and (1), one can write for this quantity

$$\Theta_z = T_z^{(-)} \sqrt[3]{\frac{5}{3} \frac{\sqrt{2\sigma}(1+\sigma)\alpha_w^2}{\pi c^2 g c_p^2 \rho^2 z} \left(1 - \frac{T_w}{T_z^{(-)}}\right)^2} \exp\left[-\frac{\sigma}{2}\left(\frac{y}{cz}\right)^2\right]. \tag{6}$$

Substituting (6) in (4), one can then obtain

$$T_z^{(-)} = \frac{[\alpha_f (T_f - T_{z\,av})\, b\,(b - \Delta b_z)^2]}{2 c_p V_f^2 \rho c z \sqrt[3]{\frac{5}{3} \frac{\sqrt{2\sigma}(1+\sigma)\alpha_w^2}{\pi c^2 g c_p^2 \rho^2 z} \left(1 - \frac{T_w}{T_z^{(-)}}\right)^2} \Phi(u_1)}, \tag{7}$$

where

$$u_1 = \frac{b - \Delta b_z}{cz} \sqrt{\frac{\sigma}{2}}; \quad \Phi(u_1) = \frac{2}{\sqrt{\pi}} \int_0^{u_1} \exp(-t^2)\, dt.$$

To calculate $T_z^{(-)}$ using (7), it is necessary to determine the flow rate of the air in the transverse section in the upward and downward flows which, according to the mass conservation law, should have the same flow rate. To determine this flow rate in a given transverse section, it is necessary to know the velocity field.

For a square horizontal cross section (Fig. 1b) for $l = 2b$,

$$V_{fz} = w_{z\,av} l (b - \Delta b_z) = l \int_0^{b-\Delta b_z} w_z dy \tag{8}$$

and, according to [7] and (1),

$$V_{fz} = l \sqrt[3]{\frac{1.2}{c} \sqrt{\frac{1+\sigma}{\pi\sigma}} \frac{g \alpha_w z}{c_p \rho} \left(1 - \frac{T_w}{T_z^{(-)}}\right)} \sqrt{\frac{\pi}{2}}\, cz \Phi(u_2). \tag{9}$$

where $u_2 = b - \Delta b_z / cz\sqrt{2} = y/\sqrt{2}\, cz$.

From the mass conservation law we have

$$V_z^{(-)} = V_z^{(+)} \quad \text{or} \quad V_{fz} = w_z^{(-)} \Delta b_z^2. \tag{10}$$

Hence,

$$w_z^{(-)} = \frac{V_{fz}}{\Delta b_z^2} = \frac{2bcz}{\Delta b_z^2} \sqrt{\frac{\pi}{2}} \sqrt[3]{\frac{1.2}{c} \sqrt{\frac{1+\sigma}{\pi\sigma}} \frac{g \alpha_w z}{C_p \rho} \left(1 - \frac{T_w}{T_z^{(-)}}\right)} \Phi(u_2). \tag{11}$$

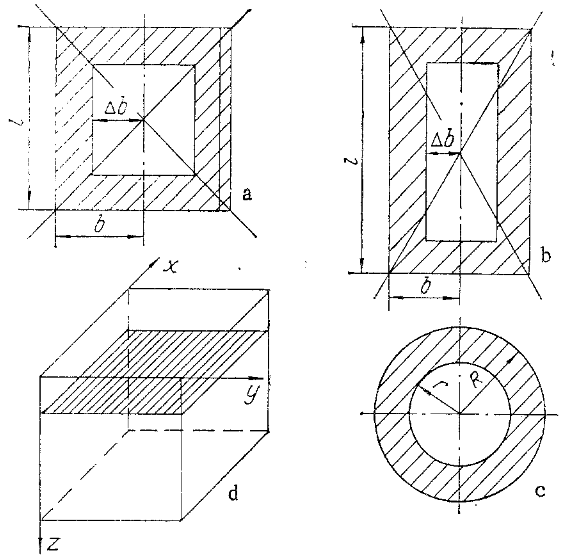

Fig. 1. Diagrams of the transverse sections: a) square; b) rectangle; c) circle; d) coordinate system and model of the channel.

However, the velocity fields $w_z^{(+)}$ and $w_z^{(-)}$ must be matched in a point where $w_z^{(+)} = w_z^{(-)}$, i.e., for such values of Δb_z and $z = h$, that this condition holds. According to [7], the velocity field near the wall is determined from the expression

$$w_z^{(+)} = \sqrt[3]{\frac{1.2}{c}\sqrt{\frac{1+\sigma}{\pi\sigma}}\frac{g\alpha_w z}{c_p\rho}\left(1-\frac{T_w}{T_z^{(-)}}\right)}\exp\left[-\frac{1}{2}\left(\frac{y}{cz}\right)^2\right]. \tag{12}$$

We then obtain from (11) and (12),

$$\exp\left[-\frac{1}{2}\left(\frac{y}{cz}\right)^2\right] = \frac{2bcz}{\Delta b_z^2}\sqrt{\frac{\pi}{2}}\Phi\left(\frac{y}{\sqrt{2}\,cz}\right). \tag{13}$$

We thus obtain a transcendental equation for Δb_z. By solving this equation graphically or by iteration, we find the required value of Δb_z, and hence easily obtain the dividing boundary of the flows.

Having determined the value of Δb_z, we find the coordinate of the point where the flow begins to turn on the line which divides the downward and upward flows. Assuming that $b - \Delta b_z = H - h$, we find graphically or numerically the values of Δb_z and h from (13) for $y = b - \Delta b_z$ and $z = H - (b - \Delta b_z)$.

One can obtain, from (7) and (9),

$$T_z^{(-)} = \frac{[\alpha_f(T_f - T_{zav})\,b\,(b-\Delta b_z)^2]}{\pi c^2 z^3\sqrt[3]{\frac{7.2\sqrt{2}(1+\sigma)^2 g\alpha_w z}{3c\pi^2\sqrt{\sigma}\,c_p\rho}\left(1-\frac{T_w}{T_z^{(-)}}\right)^4}\sqrt{\frac{\pi}{2\sigma}}\Phi^2(u_2)\Phi(u_1)}, \tag{14}$$

where

$$T_{zav} = T_z^{(-)}\left[1-\frac{cz}{b-\Delta b_z}\sqrt{\frac{\pi}{2\sigma}}\sqrt[3]{\frac{5\sqrt{2\sigma}(1+\sigma)\alpha_w^2}{3\pi c^2 g c_p^2\rho^2 z}\left(1-\frac{T_w}{T_z^{(-)}}\right)^2}\Phi(u_1)\right]. \tag{15}$$

This expression contains only one unknown ($T_z^{(-)}$) which can be determined graphically or by iteration.

TABLE 1. Calculation of the Temperature and Velocity Fields as Functions of the Configuration of the Confined Space

Configuration	Temperature $T^{(-)}$	$w_z^{(+)}$, $w_z^{(-)}$, Θ_z	Re_z
Square	282,515	$w_z^{(+)} = 0{,}234\, z^{1/3} \exp\left[-74{,}36\left(\frac{y}{z}\right)^2\right]$ $w_z^{(-)} = 0{,}024\, \frac{z^{4/3}}{\Delta b_z}\, \Phi\left(\frac{b-\Delta b_z}{0{,}116\, z}\right)$ $\Theta_z = 1{,}664\, z^{-1/3} \exp\left[-59{,}49\left(\frac{y}{z}\right)^2\right]$	$\mathrm{Re}_1 = 1622$ $\mathrm{Re}_2 = 4019$ $\mathrm{Re}_3 = 6762$ $\mathrm{Re}_4 = 9689$ $\mathrm{Re}_{4,372} = 10806$
Rectangle	282,657	$w_z^{(+)} = 0{,}236\, z^{1/3} \exp\left[-74{,}36\left(\frac{y}{z}\right)^2\right]$ $w_z^{(-)} = 0{,}024\, \frac{z^{4/3}}{\Delta b_z}\, \Phi\left(\frac{b-\Delta b_z}{0{,}116\, z}\right)$ $\Theta_z = 1{,}692\, z^{-1/3} \exp\left[-59{,}49\left(\frac{y}{z}\right)^2\right]$	$\mathrm{Re}_1 = 1628$ $\mathrm{Re}_2 = 4055$ $\mathrm{Re}_3 = 6868$ $\mathrm{Re}_4 = 9920$ $\mathrm{Re}_{4,318} = 10927$
Circle	282,588	$w_z^{(+)} = 0{,}235\, z^{1/3} \exp\left[-74{,}36\left(\frac{y}{z}\right)^2\right]$ $w_z^{(-)} = 0{,}024\, \frac{z^{4/3}}{\Delta b_z}\, \Phi\left(\frac{b-\Delta b_z}{0{,}116\, z}\right)$ $\Theta_z = 1{,}679\, z^{-1/3} \exp\left[-59{,}49\left(\frac{y}{z}\right)^2\right]$	$\mathrm{Re}_1 = 1623$ $\mathrm{Re}_2 = 4027$ $\mathrm{Re}_3 = 6793$ $\mathrm{Re}_4 = 9779$ $\mathrm{Re}_{4,352} = 10863$

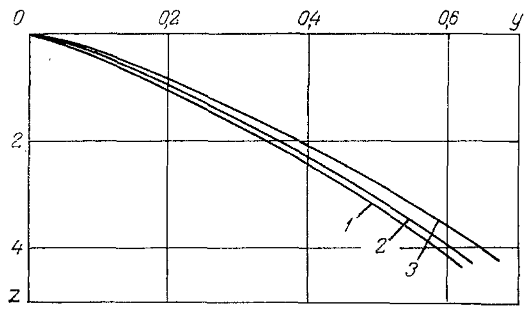

Fig. 2. Dividing line between the flows: 1) square; 2) circle; 3) rectangle. The coordinates z and y are in meters.

Equations (14) and (15) contain the heat-exchange coefficients of the air flow with the wall α_w and with the field α_f. An analysis of the data [1-4] shows that the heat-exchange coefficients should be calculated using the dependences recommended by McAdams [2], with corrections proposed by Bogoslovskii [4].

Then, according to [2], we have for a horizontal free surface

$$\mathrm{Nu} = 0.205^{*}\,(\mathrm{Gr\,Pr})^{1/3}, \tag{16}$$

and hence

$$\alpha = 0.205\left(\frac{\lambda^2 g \beta c_p \rho}{\eta}\right)^{1/3} \Delta T^{1/3}. \tag{17}$$

For the field, the coefficient α is increased on average by 25% [4], and takes the values

$$\alpha_f = 0.256\left(\frac{\lambda^2 g \beta c_p \rho^2}{\eta}\right)^{1/3} \Delta T_f^{1/3}. \tag{18}$$

*In the SI system.

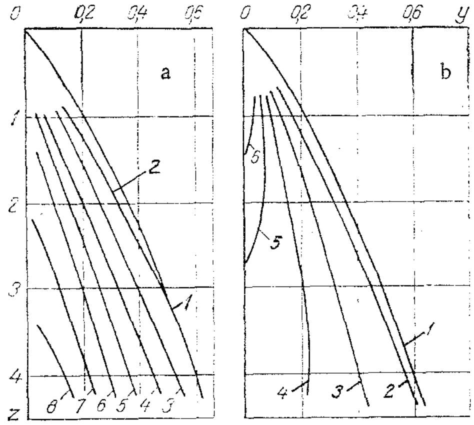

Fig. 3. Distribution of the velocity fields (a) and temperature fields (b): (a) 1) dividing line between the flows; 2, 3, 4, 5, 6, 7, 8) flow velocities w (5, 10, 15, 20, 25, 30, 35)$\cdot 10^{-2}$ m/sec, respectively; (b) 1) dividing line between the flows; 2, 3, 4, 5, 6) temperatures of the flow Θ (0.3, 0.6, 0.9, 1.2, 1.5) °K, respectively.

For the chosen direction of the flow along the wall downwards α_w is decreased by 30% in comparison with α_f [4] and is determined from the formula

$$\alpha_w = 0.179 \left(\frac{\lambda^2 g \beta c_p \rho^2}{\eta} \right)^{1/3} \Delta T_w^{1/3}. \tag{19}$$

If the horizontal section of the confined space is a rectangle then, similarly to the analysis of the square section, expression (13) takes the form

$$\exp\left[-\frac{1}{2} \left(\frac{y}{cz} \right)^2 \right] = \frac{l + 2b}{l - 2(b - \Delta b)} \; \frac{cz}{\Delta b} \sqrt{\frac{\pi}{2}} \Phi \left(\frac{y}{\sqrt{2}\, cz} \right), \tag{20}$$

and in the case of a circular horizontal section,

$$\exp\left[-\frac{1}{2} \left(\frac{y}{cz} \right)^2 \right] = \frac{R - r}{r^2} cz \sqrt{\frac{\pi}{2}} \Phi \left(\frac{y}{cz\sqrt{2}} \right). \tag{21}$$

The remaining method of calculation is the same.

By way of example of the calculation, we consider the case of a convective motion of air in cavities with square, rectangular, and circular horizontal sections. We specify the following values of parameters: H = 5.0 m, b = 5.0 m, l = 20.0 m (for rectangle), l = 10 m (for square), R = 5.0 m, λ = 0.024 W/(m$\cdot$°K), $\beta = 3.67 \cdot 10^{-3}$ °K, $\eta = 1786 \cdot 10^{-8}$ kg/(m$\cdot$sec), ρ = 1.2 kg/m^3, c_p = 1000 J/(kg$\cdot$°K), T_w = 275°K, T_f = 285°K, c = 0.082, and σ = 0.8.

To determine the dividing boundary between the flows, Eqs. (13), (20), and (21) are brought to a form which is convenient for an approximate solution (Table 1).

Figure 2 shows the dividing lines between the flows for each of the cases considered. The coordinates of the turning points for the square, rectangle, and circle have the values (0.628; 4.372), (0.682; 4.318), and (0.648; 4.352), respectively.

The temperature $T^{(-)}$ can be determined from Eq. (14) and takes on the following values: 282.5°K (square), 282.7°K (rectangle), and 282.6°K (circle).

The distributions of temperature (isotherms) and velocities (isochores) for the square section are shown in Fig. 3.

The values of the criterion Re vary from 1600 to 11,000. It takes the largest value for the rectangular section and the least value for the square section.

NOTATION

Θ_z, excess temperature in section z; Θ_{zm}, maximum excess temperature in section z; T_w, temperature of the wall; T_f, temperature of the field; $T_z^{(-)}$, absolute temperature at an arbitrary point of the section z of the upward flow; w_{zm}, maximum flow velocity in section z; $w_z^{(+)}$, $w_z^{(-)}$, flow velocity at an arbitrary point of the section z of the downward and upward flows, respectively; σ, experimental constant whose value is equal to 0.8; c, experimental constant equal to 0.082; $V_{fz}^{(+)}$, $V_{fz}^{(-)}$, flow rates of air in the downward and upward flows, respectively; g, gravity constant; c_p, heat capacity of air; ρ, density of air; λ, thermal conductivity; β, volume expansion coefficient; η, dynamic viscosity; α_w and α_f, heat-exchange coefficients of the air flow with the wall and field, respectively; b, dimensions of the volume in the y direction; H, dimensions of the volume in the z direction; h, dimension of the active region of the convective flow; Δb_z, width of the upward flow in the section z; l, dimensions of the volume in the x direction; R, radius of the confined space; r, radius of the upward flow.

LITERATURE CITED

1. M. A. Mikheev, Foundations of Heat Exchange [in Russian], Gosénergoizdat, Leningrad (1956).
2. W. H. McAdams, Heat Transmission, 3rd ed., McGraw-Hill, New York (1954).
3. A. G. Egiazarov and V. V. Karelina, "Convective heat exchange on surfaces in conditions of confined spaces (analytic review)," in: Ventilation and Air Conditioning [in Russian], Riga Polytechnic Institute (1978), pp. 53-65.
4. V. N. Bogoslovskii, Heat Regime of Buildings [in Russian], Stroiizdat, Moscow (1979).
5. G. A. Ostroumov, Free Convection in Conditions of Internal Problem [in Russian], State Technical and Theoretical Press, Moscow–Leningrad (1952).
6. G. Z. Gershuni and E. M. Zhukovitskii, Convective Stability of Incompressible Fluids, Halsted Press (1976).
7. I. A. Shelelev, Aerodynamics of Air Flows in an Enclosure [in Russian], Stroiizdat, Moscow (1978).